Selective resonant tunnelling – turning hydrogen-storage material into energetic material

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A new formula for nuclear fusion cross-sections reveals the existence of a low energy resonance in p + ^{6}Li system, and the selectivity of low energy resonance. It indicates that lithium-6 might be a nuclear fuel in condensed matter nuclear science. Evidences from both hot fusion and 'cold fusion' experiments are presented.

Keywords: Abnormal isotope abundance ratio, 3-parameter formula, low-energy resonance, proton + ^{6}Li system, selective resonant tunnelling.

Introduction – 25 years of pursuing nuclear energy without contamination

Nuclear energy is necessary to meet the world needs of energy eventually. Can we have the nuclear energy without nuclear contamination? It is possible that this problem could be solved in the paradigm of 'cold fusion' (i.e. condensed matter nuclear science (CMNS), or nuclear reaction at normal temperature, etc.). From the binding energy of nuclei, it is clear that we may explore the nuclear energy as long as we may put a nucleon (neutron or proton) into any nucleus, because the dominant nuclear force always tends to attract nucleons together when a free nucleon enters any nucleus, and the binding energy for all nuclei (both stable and unstable) is positive. The key is how to put one more nucleon into any nucleus. Putting a neutron into fissile nucleus is easy; however, to keep a self-sustaining neutron source is not so easy. On the other hand, fusion of high-temperature plasma of light nuclei is feasible; however, to keep a self-sustaining high-temperature plasma is not so easy. If we are able to guide a proton into a nucleus, then the hydrogen storage materials might be turned into an energetic material. Thus proton would act just like a neutron without the need of neutron breeding. The question is how we are able to find a nucleus with a low energy nuclear resonance level which facilitates the tunnelling of a proton through the Coulomb barrier.

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New formula of fusion cross-sections for resonance

It is impossible to find a resonance peak from the cross-section of low energy proton near thermal energy, because there are no such experimental data. However, the theory might predict the extremely low energy behaviour based on existing data. For low-energy projectile, the cross-section, $\sigma(E)$ is expressed by a phase shift of S-partial wave function, δ_0, provided that S-partial wave is dominant

$$\sigma(E) = \frac{\pi}{k^2}(1 - |e^{i2\delta_0}|^2). \qquad (1)$$

This expression does not show clearly the Gamow factor for charged particle interaction and also how the resonance would overcome the Coulomb barrier. Thus we derived another expression which is identically equal to eq. (1)[1–6]

$$\sigma(E) = \frac{\pi}{k^2}\frac{(-4W_i)}{W_r^2 + (W_i - 1)^2}. \qquad (2)$$

Here $W \equiv \cot\delta_0 \equiv W_r + iW_i$ is introduced to replace δ_0. The imaginary part, W_i, describes the absorption in the nuclear potential well. This formula clearly shows the physical meaning of a resonance: it corresponds to an energy which makes $W_r = 0$ and $W_i = -1$ (see note 1). Indeed, W is the coefficient of a linear composition of two independent solutions of the Schrödinger equation. In case of charged nuclei collision, $\phi(r) = W \cdot F_0 + G_0$, where $\phi(r)$ is the reduced radial wave function in the Coulomb field, F_0 and G_0 are the regular and irregular Coulomb wave functions respectively and r is the radial distance from the centre of nuclear potential well. At the resonance energy, $\phi(r) = (0 - i) \cdot F_0 + G_0 \xrightarrow[r\to\infty]{} e^{-ikr}$. This implies an incoming spherical wave without any reflection, or perfect absorption of incoming wave by a nuclear potential well. That is the physical meaning of a resonance. Then, where is the Gamow factor? The answer is hidden in the energy dependence of W for the charged nuclei reaction. Based on the continuity of the wave function at the interface between nuclear well and

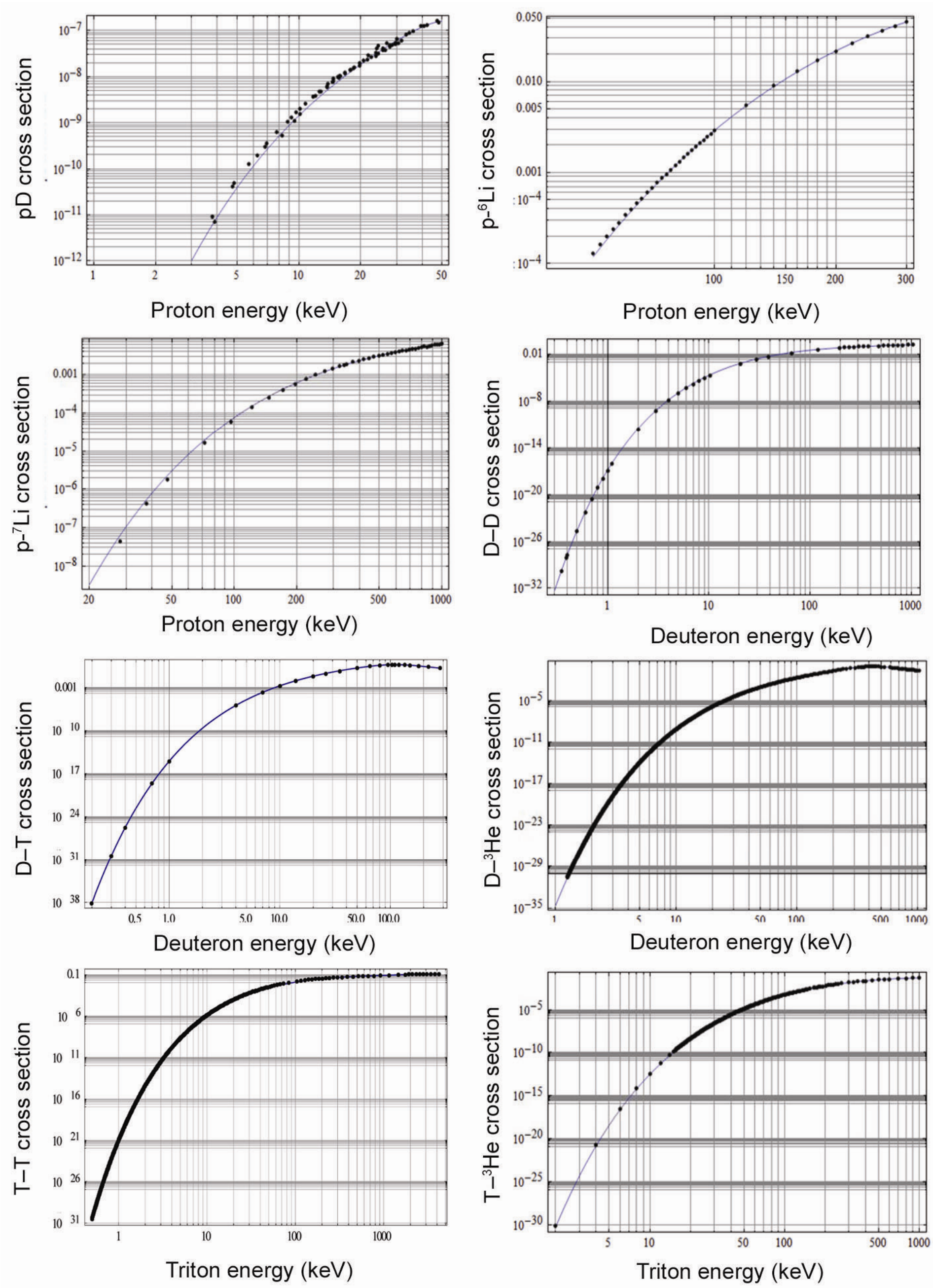

Figure 1. Comparison between experimental data points and theoretical fitting curves based on eq. (4). Cross section is in barns and energy is in the lab system.

Coulomb barrier, we may find the energy dependence of W as follows[6]

$$W = -\left(\frac{G_0}{F_0}\right)_{r=a} \left[\frac{k_1 \cot[k_1 a] - \frac{k}{G_0} \left.\frac{\partial G_0}{\partial \rho}\right|_{r=a}}{k_1 \cot[k_1 a] - \frac{k}{F_0} \left.\frac{\partial F_0}{\partial \rho}\right|_{r=a}} \right], \tag{3}$$

where k_1 and k are the wavenumbers in the nuclear potential well and in the Coulomb field respectively, $\rho = k \cdot r$ and a is the radius of the nuclear potential well. Based on the energy dependence of F_0 and G_0, we may separate W into two factors in eq. (3): the fast varying factor in the first bracket and the slow one in the second bracket. Since G_0 is exponentially rising and F_0 is exponentially decreasing when r is approaching the nuclear boundary, a, the ratio of

$$\left(\frac{G_0}{F_0}\right)_{r=a} \propto \left(\frac{e^{2\pi/(ka_c)} - 1}{2\pi}\right) \equiv \theta^2,$$

is an extremely large factor at low energy

$$a_c \equiv \frac{4\pi\varepsilon_0\hbar^2}{Z_a Z_b \mu e^2},$$

where ε_0 is the vacuum dielectric constant, $\hbar$ the Planck constant divided by 2π, e the charge of the proton, μ the reduced mass, and Z_a and Z_b are charge numbers of the colliding nuclei respectively. Therefore, we may assume

$$W = \theta^2(w_{\rm r} + {\rm i}w_{\rm i}) = \theta^2(C_1 + C_2 E_{\rm lab} + {\rm i}w_{\rm i}),$$

and this leads to the expression with Gamow factor ($1/\theta^2$), explicitly[7]

$$\sigma(E) = \frac{\pi}{k^2}\frac{1}{\theta^2}\frac{(-4w_{\rm i})}{w_{\rm r}^2 + \left(w_{\rm i} - \frac{1}{\theta^2}\right)^2}$$

$$\approx \frac{\pi}{k^2}\frac{1}{\theta^2}\frac{(-4w_{\rm i})}{(C_1 + C_2 E_{\rm lab})^2 + \left(w_{\rm i} - \frac{1}{\theta^2}\right)^2}. \quad (4)$$

This assumption is supported by experimental data for eight major fusion cross-sections: p + D, p + ^{6}Li, p + ^{7}Li, d + D, d + T, d + ^{3}He, t + T and t + ^{3}He (Figure 1). (Logarithmic scales are used to show the good fit in very low energy region; however, the usual resonance peaks for d + T and d + ^{3}He curves become flat in this scale.)

In Figure 1, the solid lines are the fitting curves using eq. (4) with three parameters: C_1, C_2 and $w_{\rm i}$. The dots are experimental data points from National Nuclear Data Center (NNDC) in Brookhaven National Lab[8]. Using the least squares method we may find three parameters for each reaction, as shown in Table 1.

The derivation of this three parameter equation (eq. (4)) does not invoke 'compound nucleus model'; hence, it contains not only the conventional Gamow factor ($1/\theta^2$) at front, but also has an energy dependence of ($1/\theta^2$) in the *S*-factor (or the astrophysical function). This unique feature is in good agreement with experimental data using only three parameters, while the Naval Research Laboratory (NRL) formula in the famous *Plasma Formulary Handbook*[9,10] failed to fit these experimental data even if five parameters were introduced[6]. Because NRL-formula tried to use polynomials only to approximate an exponential dependence on energy, the failure was inevitable. This new formula for cross-section even corrected a set of misleading data points in the early NNDC d + T fusion cross-section[3]. NNDC did not notice these mistakes until this formula was published in 2002. The most important feature of this new formula is to provide a tool for searching the low energy resonance. According to eq. (2), resonance would appear at $W_{\rm r} = 0$, i.e. $C_1/C_2 = 0$. In Table 1, among the 8 fusion data, only p + ^{6}Li cross-section data might be fitted by three parameters with $C_1 = 0$. Thus, the 'hot fusion' data imply a low-energy resonance only in the p + ^{6}Li system. Then, it is interesting to see what we have observed in early 'cold fusion' experiments.

Evidences for lithium-6 depletion

Twelve years ago, T. Passell[11] a senior nuclear physicist, did a series of TOF-SIMS analyses for Pd samples exposed to gaseous hydrogen and deuterium. Most of the samples from Japan, the US, and China show an abundance ratio (^{7}Li/^{6}Li) > 12.56 (the terrestrial value). The 'Tsinghua University sample E' has the highest ratio of

Table 1. Three parameters for eight fusion reactions

Reaction	C_1	C_2 (1/keV)	$w_{\rm i}$	Norm/number of data points (Barn)	[Cross-section]$_{\rm max}$ (Barn)	[Energy]$_{\rm max}$ (keV)
d + T	0.544	–0.00558	–0.390	0.227/24	5.0	280
d + ^{3}He	1.13	–0.00304	–0.670	0.0520/800	0.8	1034
d + D	4.78	–0.00226	–0.186	0.00567/39	0.177	1045
t + T	36.8	–0.00928	–24.6	0.0129/757	0.115	4300
t + ^{3}He	2.79	0.000959	–1.04	0.00331/225	0.0214	1000
p + D	8.04×10^7	-1.80×10^6	-5.31×10^7	3.35×10^{-8}/74	1.45×10^{-7}	48.1
p + ^{6}Li	0	–0.00818	–6.14	0.00493/41	0.063	400
p + ^{7}Li	30.9	–0.00367	–4.18	0.000310/42	0.00633	998

Table 2. Evidences for lithium-6 depletion

Sample designation	^{7}Li/^{6}Li ratio	Uncertainty	One-sigma range
Li Tsinghua sample E	23.3	1.8	21.5–25.1
Li Tsinghua sample D	13.1	1.1	12.0–14.2
Li Tsinghua sample B (Virgin)	12.9	0.8	12.1–13.7

23.3, while for the virgin sample it is 12.9 (Table 2). It was a palladium foil sample exposed mainly to hydrogen gas (deuterium appeared as a natural isotope). This anomaly was confirmed by later TOF-SIMS analysis in China Institute of Atomic Energy with the ratio depth profile on the foil surface. The observed depletion of ^{6}Li is supporting evidence for the proposed existence of a low energy resonance in p + ^{6}Li system.

The new features of selective resonant tunnelling in metal-hydrides

The derivation of eq. (2) does not invoke the compound nucleus model, and the tunnelling process is no longer separated in to two independent steps. Therefore, selective resonant tunnelling in metal-hydrides has its new features.

The selectivity in reaction channel

When the resonance energy is low, the Gamow penetration factor $1/\theta^2$ is an extreme small number. In order to show the peaked feature of a resonance in eq. (4), the imaginary part of the nuclear potential well must satisfy $w_\mathrm{i} \approx -1/\theta^2$. It corresponds to a very slow nuclear reaction inside the nuclear well, because

$$|w_\mathrm{i}| = \frac{|W_\mathrm{i}|}{\theta^2} \approx |\mathrm{Im}[k_1 a_\mathrm{c} \cot[k_1 a]]| < |\mathrm{Im}[k_1 a_\mathrm{c}]|$$

$$\approx \left| \sqrt{\frac{2\mu(E-U_\mathrm{r})}{\hbar^2}} \left(\frac{-U_\mathrm{i}\ a_\mathrm{c}}{2(E-U_\mathrm{r})} \right) \right|$$

$$\approx \left| \frac{-U_\mathrm{i}/\hbar}{\dfrac{\sqrt{2\mu(E-U_\mathrm{r})}}{\mu a}} \frac{a_\mathrm{c}}{a} \right| \approx \frac{1/\tau_\mathrm{life}}{1/\tau_\mathrm{bounce}} \approx \frac{\tau_\mathrm{bounce}}{\tau_\mathrm{life}}. \quad (5)$$

where τ_bounce is the time for wave bouncing back and forth in the nuclear potential well which is in the order of 10^{-23} sec, $\tau_\mathrm{life} = -\hbar/U_\mathrm{i}$ is the life-time of the wave inside the nuclear potential well. The criterion $w_\mathrm{i} \approx -(1/\theta^2)$ (i.e. $\tau_\mathrm{life} \approx \theta^2 \tau_\mathrm{bounce}$) implies an extremely long life-time – a very slow nuclear reaction rate! Consequently, the low energy resonant tunnelling is only effective for a weak interaction[12]. Strong nuclear interaction or electromagnetic interaction is too strong to have any resonant tunnelling effects at low energy, because the life time for strong nuclear interaction is in the order of 10^{-23} sec, and the life time for electromagnetic interaction is in the order of 10^{-17} sec. Both of them cannot satisfy the criterion of $w_i \approx -(1/\theta^2)$. Indeed, the nuclear reaction acts like a damping to wave (absorption or attenuation). A strong damping would stop the propagation of a wave, and kill any resonant tunnelling. This is the selectivity in the resonant tunnelling at low-energy. No neutron emission or strong Gamma ray would be accompanied with a low-energy resonant tunnelling[13]. This conclusion is very different from that of the 'compound nucleus model' which predicts the decay of the compound nucleus through preferably the fastest reaction channel; however, selectivity of the resonant tunnelling selects the slow reaction channel instead. This is understandable, because in the case of light nucleus, there is not enough collisions to make injected projectile to forget its 'history' and decay independent of its 'history'. The 'compound nucleus model' is no longer valid here. What formed in the metal hydrides is a composite state (i.e. a sinusoidal wave in nuclear potential well is connected to a wave function $\phi(r) = W \cdot F_0 + G_0$ in a screened Coulomb field to keep memory of all phase information of the wave), but not a 'compound nucleus' which has no memory of incoming wave.

The discrete energy level in the metal-hydride

The long life time of the composite state means a very narrow energy level. In the beam-target experiments, it implies a very little occupancy of incoming beam at this energy level when the width of the beam energy is much greater than the width of the resonance energy level. However, the discrete energy level in metal hydride is very different from the continuum of an injected beam. When metal hydride transforms from α-phase to β-phase, a macroscopic number of protons are occupying the discrete energy level, no matter how narrow the energy level is. In the case of beam-target experiments, the integral over energy distribution of a beam is usually applied to obtain the total probability of resonant tunnelling; therefore, the result would be almost same no matter how sharp the resonance is, if a uniform distribution in beam energy width is assumed. Nevertheless, the discrete energy level in metal hydride would have different probability of tunnelling through the Coulomb barrier when the energy level is tuned into the resonance peak.

Lithium-6 enriched metal-hydride is a good additive for CMNS experiments

Lithium was widely used in early CMNS experiments as an additive following Fleischmann and Pons; however, the lithium-6 abundance was not mentioned. The hot fusion data might guide us to solve the reproducibility problem in early CMNS experiments using lithium-6-enriched additives.

Note

1. From eq. (2), it is easy to think that a resonance would appear when $W_\mathrm{i} = +1$. Indeed, W_i must be a negative number due to the absorption

inside the nuclear potential well. From eq. (3) in ref. 4, we have $W_i = \theta^2 \mathrm{Im}[k_1 a_c \cot[k_1 a]]$; here

$$k_1 = \sqrt{\frac{2\mu}{\hbar^2}(E - (U_r + iU_i))},$$

is the wavenumber inside the nuclear potential well. When fusion reaction appears inside the nuclear potential well, the nuclear potential becomes a complex number ($U_r + iU_i$) and it has an imaginary part, $U_i \cdot U_i < 0$ corresponds to an absorption (fusion reaction reduces the amplitude of the wave function). Hence, the imaginary part of the wave number

$$\mathrm{Im}[k_1 a_c] \approx \sqrt{\frac{2\mu(E - U_r)}{\hbar^2}}\left(\frac{-U_i a_c}{2(E - U_r)}\right) > 0.$$

However, near the resonance ($W_r = 0$) we must have $\mathrm{Re}[\cot[k_1 a]] < 0$, in order to have a smooth connection of wave function to G_0 (eq. (3)); therefore, W_i must be a negative number in a real resonance. This can be seen also from the 6th line under the eq. (2). Only if $W_i = -1$, there will be an incoming spherical wave (e^{-ikr}) that corresponds to a perfect absorption. If $W_i = +1$, there will be an outgoing spherical wave only (e^{+ikr}) that does not correspond to a resonant absorption.

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